

Two day's worth of KATRIN data

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The KATRIN experiment plans to take data for 3 to 5 years so as to achieve a ten-fold improvement on previous experiments in the value for the electron antineutrino effective mass. As shown elsewhere this experiment can also test a 3 + 3 model of the neutrino mass states with three unconventionally large masses. It is shown here that only two day's worth of data should be more than sufficient to exclude or validate this model. It is further shown that while final state distributions probably cannot explain the good fits this model gave to previous direct mass data, they would alter the predictions of the 3 + 3 model for KATRIN, without making them less observable. Four further considerations are discussed which strengthen the case for taking the unconventional model more seriously.

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I. INTRODUCTION

The KATRIN tritium β -decay experiment expects to set an improved upper limit of 0.2 eV on the $\bar{\nu}_e$ effective mass, $m_\nu(\text{eff})$ after 3 years of data taking, or it might discover the actual value at a 5σ level if $m_\nu(\text{eff}) > 0.35$ eV. [1] In single β -decay the effective mass can be defined in terms of the separate mass states having masses m_j contributing to the $\bar{\nu}_e$ state: [2]

$$m_\nu^2(\text{eff}) = \sum_j |U_{e,j}|^2 m_j^2 \quad (1)$$

To find a value for $m_\nu(\text{eff})$ from the tritium β - spectrum one can do a fit of the distribution of the electron kinetic energies, E , near the endpoint E_0 , using the phase space term, i.e., the square of the Kurie function $K(E)$ where: [3]

$$K^2(E) = (E_0 - E) \sqrt{R((E_0 - E)^2 - m_\nu(\text{eff})^2)} \quad (2)$$

and where R is the Ramp function defined by $R(x) = x$ for $x \geq 0$, and $R(x) = 0$ for $x < 0$. Note that here E_0 is understood to be given by the decay Q-value, which is the true spectrum endpoint only when one ignores final state distributions, and the acceptance of an experiment for energies close to E_0 . Moreover, fits using Eq. 2 are only appropriate if the separate m_j are not distinguishable experimentally. Otherwise, the proper procedure is to fit the spectrum near E_0 using $K^2(E)$ defined as: [3]

$$K^2(E) = (E_0 - E) \sum |U_{e,j}|^2 \sqrt{R((E_0 - E)^2 - m_j^2)} \quad (3)$$

which essentially describes a sum of spectra for each of the m_j with weight $|U_{e,j}|^2$. Eq. 3 has been used, for example, in searches for a possible sterile mass state. [4–7]

Another use of Eq. 3 was made by the author in ref. [8] in testing his 3 + 3 model of the neutrino mass states consisting of three L-R doublets having unconventionally large masses. In that paper it was shown that this model gave excellent fits to the data from three existing high precision tritium β - decay experiments done by the Troitsk, Mainz and Livermore groups without necessarily being in conflict with the cosmological constraint on Σm or oscillation data [8]. Interestingly, those good fits to the model were obtained despite the 95% C.L. upper limit $m_\nu(\text{eff}) < 2$ eV [2] two of those experiments have set on $\bar{\nu}_e$.

II. THE 3 + 3 MODEL

The 3+3 model postulates three ν_L, ν_R doublets having masses that were originally inferred from an analysis of SN1987A data [9–11]. The model was given subsequent support by dark matter fits to the Milky Way galaxy and to four galaxy clusters. [12] The masses of the three doublets are: $m_1 \approx m_2 = 4.0 \pm 0.5$ eV, $m_3 \approx m_4 = 21.2 \pm 1.2$ eV, and $m_5^2 \approx m_6^2 = -0.2$ keV², or $\mu_5 \approx \mu_6 = \sqrt{-m_5^2} = 447$ eV. Each of the three masses would, in accordance with Eq. 3, create a specific feature in the spectrum.

The three features include: (a) a kink at $E_0 - E = m_1$, (b) a second kink at $E_0 - E = m_3$, and (c) a linear variation of $K^2(E) \propto \mu_5(E_0 - E)$ between that second kink and E_0 . As noted in ref. [8] the only detectable effect on the spectral shape given the limited statistics of existing data sets is the first kink occurring at $E_0 - E = m_3 = 21.2$ eV. In fact, this feature was found to exist in all three data sets examined with a common spectral weight $U_{34}^2 \equiv |U_{e3}|^2 + |U_{e4}|^2 = 0.5$.

A. Why revisit the model here?

The purpose of the present paper is to consider six topics that were not addressed previously in ref. [8], each

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of which will be seen to strengthen the case for taking the unconventional $3 + 3$ model more seriously. Thus, it is shown here that:

1. The good fits the model gave to three existing tritium β -decay data sets are very likely not an artifact resulting from final state distributions (FSD's).
2. These FSD's alter the predicted $3 + 3$ model results for KATRIN without making them any less observable.
3. A less radical alternative explanation to the good fits the $3 + 3$ model gave to three tritium β -decay experiments, i.e., an $m \approx 20$ eV sterile neutrino, is implausible.
4. While the Troitsk collaboration has retracted its controversial “step” in the spectrum [13], thereby raising serious questions about its inclusion in ref. [8], there are nevertheless grounds for regarding the report of its demise as premature.
5. The need for a precise understanding of the value of all systematic uncertainties (vital for KATRIN's intended goal) is significantly less important if the data should be in accord with the $3 + 3$ model.
6. KATRIN will have the sensitivity to look for all three parts of the $3+3$ model signature, and in fact will be able it to validate or reject the model in a mere two days of data-taking.

B. How FSD's might create a kink near 20 eV

Since publication of ref. [8], it was suggested to the author [14] that the good fits of the $3 + 3$ model to β -decay data might be explained due to the presence of FSD's that will distort the observed spectrum. The two types of final state distributions considered here include molecular excitations, and scattering processes in the source which lead to β -electron energy loss. Both processes lead to a modification of the endpoint region of the spectrum, but they have a very different nature. The former includes possible final states of a diatomic molecule containing tritium, i.e., T_2, HT, DT going to HeT^+, HHe^+, DHe^+ , which can be populated before beta decay, and which change the emitted β electron energy distribution from what is found when assuming a decay Q-value for a T_2 molecule in its ground state.

The latter process lowers the observed energy of electrons after emission, and the distribution of losses depends on the number of scatterings. The distribution of electron energy loss in single inelastic scattering $f_1(E)$ is the same for all experiments using gaseous tritium – see fig. 1 (b). However, the distribution of losses for any number of scatterings, i.e., $F_{Loss}(E) = \sum P_n f_n(E)$ shown in fig. 1 (c) for KATRIN is experiment dependent

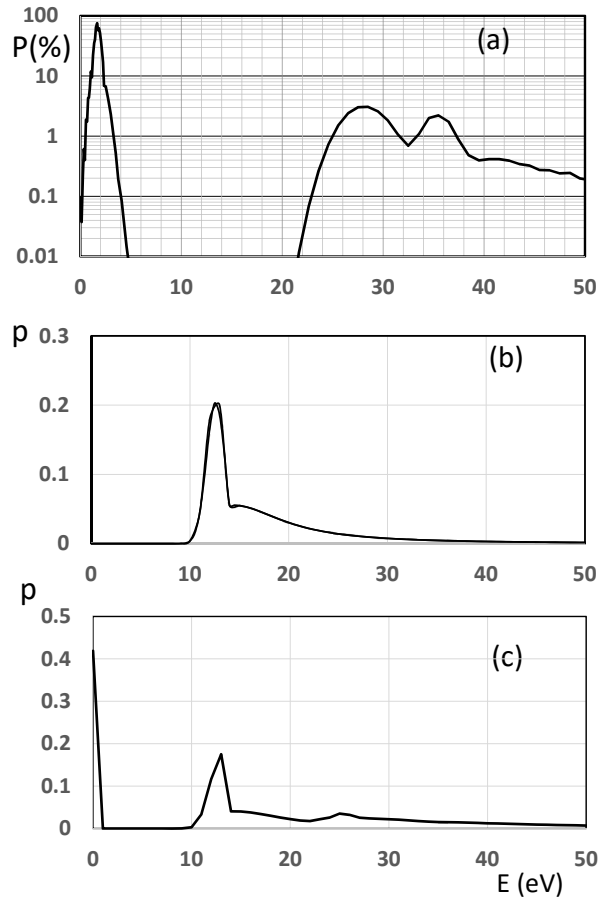


FIG. 1: **Final state distributions (FSD's)** (a) shows the distribution of molecular energy levels in tritium gas according to Saenz et al. [15], (b) shows the $f_1(E)$ distribution of energy losses by electrons in single inelastic scattering, and (c) shows the $F_{Loss}(E)$ distribution of energy losses for any number of scatterings expected in the KATRIN experiment.

because the probabilities P_n for n scatterings depend on the average column density of radioactive tritium gas, i.e., the number of mean free paths, N according to the Poisson distribution:

$$P_n = \frac{N^n e^{-N}}{n!} \quad (4)$$

The sharp peaks in Figs. 1(b) and (c) at $E_{Loss} \approx 12$ eV, and the much smaller one at $E_{Loss} \approx 25$ eV in fig. 1(c) are due to single and double inelastic scatterings, respectively. The presence of the small 25 eV peak and the broader one in fig. 1(a) at a slightly higher energy, suggest the possibility of one or both of these peaks being the true cause of the good fits reported in ref. [8] rather than the $3 + 3$ model. [14] In order to test this possibility we have done a simulation and generated spectra for both the $3+3$ model and the standard all $m_j = 0$ or $m_\nu(\text{eff}) = 0$ case using the following 5-step procedure:

1. Using the expected value of N for a particular ex-

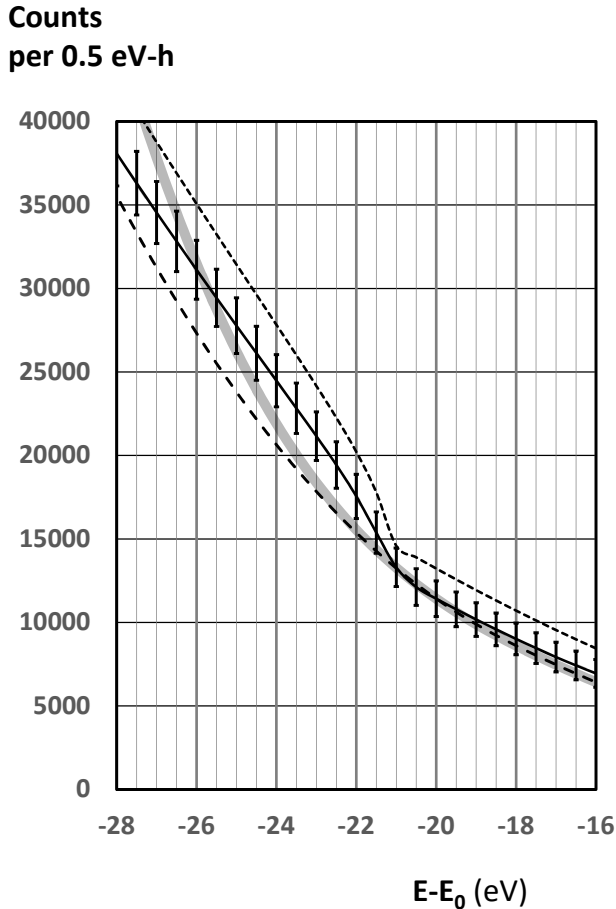


FIG. 2: **Simulated KATRIN data based on 3+3 model** for a day of data-taking, i.e., one hour of data-taking for each of 24 energy bins of 0.5 eV width. The solid curve (with 10X inflated error bars for clarity) is the expected count rate above background. The long dashed curve, which has the same count rate at 20 eV, shows the “kink-free” all $m_j = 0$ case after FSD’s and energy resolution have been included. The short dashed curve shown with slightly larger normalization (for clarity) is the 3+3 model prediction before including FSD’s and energy resolution. The thick grey curve represents a failed attempt to mimic the 3+3 model results based on an anomalous energy loss distribution including a large amplitude δ -function, as discussed in the text.

periment, find the P'_n s using Eq. 4

2. Do a convolution of the original spectrum defined by Eq. 3 with the T_2 excitation probability to find an “excitation modified” β - spectrum.
3. Find the inelastic scattering distributions, $f_n(E)$ corresponding to n scatterings by convolving the single scattering function with itself n times, and then find the resultant energy loss function $F_{Loss}(E)$ for $n \geq 0$ scatterings
4. Do a convolution of the excitation modified spectrum with $F_{Loss}(E)$ to yield the FSD-modified

spectrum $F_{FSD}(E)$.

5. Do a convolution of $F_{FSD}(E)$ with a Gaussian-distributed resolution function having an appropriate FWHM to yield a final spectrum that incorporates the effect of FSD’s and resolution.

C. Simulation results for KATRIN

The results of this simulation are shown in two figures in order to display better the distortions of the spectrum expected in two spectral regions surrounding each kink. It can easily be seen in fig. 2 that the impact of FSD’s and resolution on the spectrum is to round off the abruptness of a kink, without making it less observable. The long dashed curve in Fig. 2 (the FSD-modified spectrum for the all $m_j = 0$ case) shows that while FSD’s clearly do alter the spectrum shape near 20 eV they do not create a kink in the spectrum there when one is not originally present, i.e., when the initial spectrum is parabolic. In fact the FSD-modified long dashed spectrum shows no observable inflection at the position of the kink and it can be approximated for the last 30 eV of the spectrum by a single power law of the form $dN/dE \propto (E_0 - E - 1.0)^\gamma$ with $\gamma \approx 2.8$

In order to carry out steps 1-5 of the simulation one needs a value of N for a given experiment, which depends in part on whether a magnetic trap is used. In such a case only electrons which start with a certain range of pitch angles can pass the trap. [16]. This is less of a problem in KATRIN than some previous experiments in light of its good angular acceptance – see Fig. 9 in ref. [1], which shows transmissions as high as a constant 37% for electron energies more than 1 eV from the spectrum endpoint. KATRIN also has the advantages over earlier experiments of about four times better energy resolution (0.93 eV) and a very low background rate close to the endpoint. The expected background ($< 10^{-2}$ cps), [1] is also a significant improvement over earlier experiments such as Mainz by factors of 1.5 to 2.5. [17] Finally, we note that KATRIN has a far more sophisticated method of finding the impact of electron energy loss on the spectrum than used here. By doing a three-dimensional simulation of the motion of individual electrons emitted at various points in the gas with various directions and energies, KATRIN can find the number of mean free paths N for them to reach the detector, assuming they make it past the trap.

D. What sort of FSD might mimic the 3+3 model?

Finding an average N in a given experiment which uses a magnetic trap is difficult without doing the kind of three-dimensional simulation just described. Thus, instead of doing unrealistic simulations for the three experiments examined in ref. [8], we instead consider what

conceivable $F_{Loss}(E)$ might create a kink in the spectrum near 20 eV roughly resembling that produced by the 3+3 model, and how those results compare with those for KATRIN.

It is clear that given the existence of a sharp peak in fig. 1 (c) at $E = 0$, a $F_{Loss}(E)$ distribution with a second sharp peak near 20 eV will result in a kink of some magnitude in the final convoluted distribution near $E_0 - E = 20$ eV. Let us therefore assume an energy loss curve having a delta function of height A at 20 eV atop the $E_{Loss}(E)$ curve in fig. 1(c), and consider what values of A could produce a kink of the magnitude created by the 3 + 3 model simulation. The result of such an exercise is shown by the thick semitransparent grey curve in fig. 2, where a detectable kink occurs at -20 eV for a suitably large A value. However, not only is the kink less noticeable the one for the 3 + 3 model, but the shape of the spectrum to the left of it is quite different, being concave up rather than down as that model yields. In addition, the value of A used to create the small kink is equivalent to an impossibly large $P_2 = 220\times$ the height of the double scattering peak or 7.7 (i.e., 770%), at least when using KATRIN values for the P'_n s.

In light of this result one might question whether using P'_n s from other experiments might also require an impossibly large double scattering peak, i.e. one large enough to mimic that of the 3 + 3 model. The magnitude of the spectral kink produced from the two delta functions obviously would be proportional to the height of each one, and hence the product, P_0P_2 . We may therefore use $\frac{d(P_0P_2)}{dN} = 0$, to find N_{max} which yields a maximum possible amplitude kink. The result is $N_{max} = 1$, which yields $P_0 = 2P_2 = e^{-1} = 0.37(37\%)$, and $(P_0P_2)_{max} = 0.069$.

Using the KATRIN value $P_0 = 0.43$ [18], we find $N = 0.84$, $P_2 = 0.15$ so that $(P_0P_2)_{KAT} = 0.066$. In other words, we have shown $(P_0P_2)_{KAT} \approx (P_0P_2)_{max}$. Since the KATRIN N-value is so close to yielding the maximum amplitude kink based on the two delta function model, we can infer that what is true for that experiment is also true for others. Namely, if an impossibly large P_2 is required to produce a kink that mimicks of the 3 + 3 model for KATRIN, then the same would be true for other experiments.

E. Two day's worth of KATRIN data

KATRIN will measure the number of counts $N(E)$ in some time interval Δt integrated over the energy region from E to E_0 . Thus, two such integral measurements are needed to find the differential intensity $\Delta N/\Delta E$ for a single energy bin ΔE . Experimenters must therefore choose carefully what region of the spectrum to measure, and how much data to take for each energy bin. Apparently KATRIN intends to take data mainly over last 20 eV in light of the desire to minimize the effects of FSD's, [14] and because this is where the effects of a finite $m_\nu(\text{eff})$ are most important.

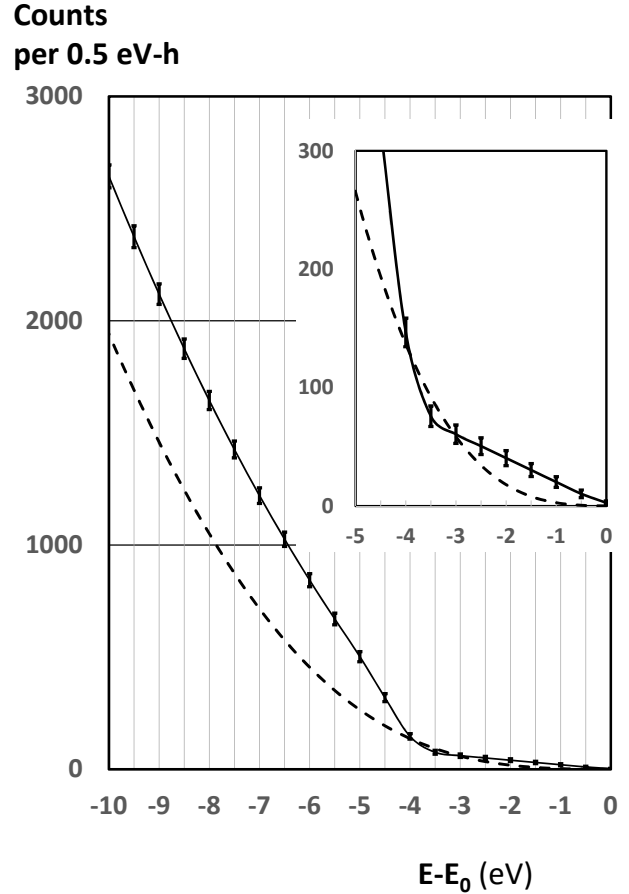


FIG. 3: **Simulated KATRIN data based on 3+3 model** for a second day of data-taking in the region $E_0 - E < 10$ eV for one hour of data-taking for each of 24 energy bins of 0.5 eV width. The solid curve (with non-inflated error bars) is the expected count rate above background for one hour of data-taking for each of 24 energy bins of 0.5 eV width. It has been normalized to yield the expected count rate at $E_0 - E = 20$ eV. The dashed curve with the same normalization shows the “kink-free” all $m_j = 0$ case after FSD's and energy resolution have been included. The insert shows the last 5 eV with an expanded vertical scale.

Of course, taking data only for $E_0 - E < 20$ eV would exclude the energy where the first and most prominent 3 + 3 model kink is predicted. It is therefore important to note that based on the simulation it should be possible to either confirm or reject the model using a mere two day's worth of data. That period would include 24 hours for each of about 24 energy bins $\Delta E = 0.5$ eV wide centered on each of the two expected kinks. In generating simulated KATRIN data we have assumed an energy resolution of 0.93 eV, a $P_0 = 43\%$, and a count rate of .01Hz for the last 1eV, [1, 18] which yields about 11,500 counts per hour in a 0.5 eV wide bin at $E - E_0 = 20$ eV.

F. The $m^2 < 0$ spectral feature in the last 4 eV

The most prominent features of the simulated 3 + 3 model results for KATRIN are the two rounded kinks at -21 eV and -4 eV. In addition, fig. 3 also shows the feature associated with the $m^2 < 0$ mass state. We assumed earlier that Eq. 3 applies regardless of the sign of m^2 . In that case for the $m^2 < 0$ state Eq. 3 yields a linear fall of the spectrum beyond the 4 eV kink, i.e., $K^2(E) \propto \mu_5(E_0 - E)$, whose slope should be in accord with the value of $\mu_5 = \sqrt{-m_5^2}$ specified by the 3 + 3 model.

However, there is a different way the $m^2 < 0$ mass state might reveal itself in the event that Eq. 3 is inappropriate for tachyons. Ciborowski and Rembielinski [19] (C-R) and more recently Radzikowski [20] have developed causal theories of tachyons which require the existence of a preferred frame of reference, perhaps that of the cosmic neutrino background (CNB). Let us assume the lab frame is close to being that preferred reference frame, which is certainly true for the CNB frame assumed to be the same as the CMB, for which $v/c = 0.00123$. In that case C-R predict that the $m^2 < 0$ component of the spectrum would have virtually constant height close to the spectrum endpoint. [19] Clearly, either possibility for the $m^2 < 0$ state, i.e., a linear fall-off as depicted or a constant height beyond 3.5 eV as C-R predict should be readily distinguishable from the quadratic fall off expected for the all $m_j = 0$ case (the dashed curve) – see fig. 3. There is, however, an interesting difference between the two possibilities, should one of them be observed (thereby confirming the existence of a tachyonic mass state). Finding that the fall-off is linear would simultaneously reveal the actual mass of the state and disprove the C-R causal theory, while finding that there is no fall-off would validate that theory but not yield a mass value.

G. What about systematic uncertainties?

KATRIN has made a major effort to understand all possible sources of systematic uncertainty. Thus, in ref. [1] they group sources of systematic uncertainty in four broad categories, having mostly to do with the tritium source. They further estimate the total systematic uncertainty in the effective mass squared of $\bar{\nu}_e$ as $\sigma_{syst} \leq 0.017 \text{ eV}^2$, or about the same as the statistical uncertainty after 3 y of data taking. [1] Let us here assume that the KATRIN results closely resemble those of the 3 + 3 model in figs. 2 and 3 after two days of data taking. If one does a one parameter fit of those results to the standard all $m_j = 0$ curve, adjusting only its normalization one finds $\chi^2 = 6880$ with 45 dof, thereby allowing this null hypothesis to be rejected with extreme confidence. This assertion depends on the use of $\sigma_{stat} = \sqrt{n}$, and it ignores systematic uncertainties σ_{syst} , which are normally combined in quadrature with σ_{stat} ,

i.e., $\sigma_{tot} = \sqrt{\sigma_{syst}^2 + \sigma_{stat}^2}$. The ignoring of systematic uncertainties is justified here because if $\sigma_{stat} \approx \sigma_{syst}$ for 3 y of data taking then $\sigma_{stat} \gg \sigma_{syst}$ when the data taking period is only 2 d. As a result it can be said that if the data closely resembles the 3 + 3 simulation results for 2 d of data taking, the null hypothesis (all very small m_j) could be rejected with very high confidence.

H. A sterile ν alternative to the 3 + 3 model?

The existence of a possible sterile neutrino ν_4 of mass $m \approx 20 - 30$ eV would seem to be a much less extreme alternative to the 3+3 model. However, while a 20 eV sterile neutrino could explain the good fits observed for three tritium decay experiments, as well as the earlier *SN1987A* data and the dark matter fits, there are some serious obstacles to this alternative:

1. The three fits to tritium experiments required a large degree of mixing ($\approx 50\%$) of the 20 eV state with the other states, and this would be ruled out by oscillation data, assuming a standard mass hierarchy.
2. A 20 eV sterile neutrino could conceivably yield results similar to the 3 + 3 model results in Fig. 2, but of course it would not agree with those in Fig. 3, with the kink near 4 eV should it be observed.
3. Based on Eq. 1 the 2 eV upper limit on the $\bar{\nu}_e$ effective mass, and a much smaller limit from cosmology on the sum of the *flavor* state masses, only the existence of a $m^2 < 0$ mass state could keep $\bar{\nu}_e$ effective mass very small. [8]

If the actual KATRIN data should closely resemble the simulated results for the 3 + 3 model in figs. 2 and 3, experimenters would be severely challenged to explain them in terms of FSD's or other systematic effects and the standard view of small neutrino masses.

Acknowledgment

The author is indebted to Hamish Robertson for calling attention to the possibility that the good fits reported in ref. [8] might be due to FSD's rather than the 3 + 3 model, and for his critical comments on an earlier version of this manuscript.

Appendix A: Is the Troitsk anomaly dead?

The inclusion of the 1999 Troitsk data in ref. [8] with its controversial “step” might be challenged, because in

a 2012 reassessment [13], after failing to explain this feature for over a decade, the Troitsk authors upon dropping some uncalibrated runs, concluded that “within the existing statistical errors, there are no reasons for introducing such a feature.” [13] In fact, one of the Troitsk authors has gone even further. V. S. Pantuev, in a private communication, has noted that Troitsk no longer bothers to display the actual spectrum because: “In [the] new analysis this step is absolutely invisible by eye in the spectrum.” [21]. Can this really be the case? While the reanalyzed Troitsk data did result in a 25% smaller am-

plitude step than originally seen, the grounds for dropping this feature are not as clear as Troitsk suggests. As can be seen in Fig. 8 of the 2012 reassessment [13], given the error bars at various times of the year the average step amplitude can be computed as $2.18 \pm 0.48 \text{ mHz}$. This result may fall short of the 5σ criterion for a discovery, which perhaps would justify Troitsk ridding themselves of their unexplained anomaly on statistical grounds. Nevertheless, a $2.18/0.48 = 4.5\sigma$ step in the spectrum is hardly insignificant, and it would be very surprising if such an effect could not be seen in the spectrum.

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